Quiz 9, Linear

2:25
2:29
4
Give 15 minutes.

Name: Key

1. (4 points) A homogeneous system of nine linear equations in six unknowns has two fixed solutions that are not scalar multiples of one another, and all other solutions are linear combinations of these two solutions. Can the set of all solutions be described with fewer than nine homogeneous linear equations? If so, how many?

$$\begin{array}{cc} & 6 \\ 9 & \begin{bmatrix} A & & \end{bmatrix} \end{array}$$

Two solutions which are not multiples

$\Rightarrow \dim(\text{Nul } A) = 2$

$\Rightarrow \dim(\text{Col } A) = 4 = 6 - 2$ ← n ← dim(Nul A)

$\boxed{\text{So only } \underline{4} \text{ equations are necessary.}}$

$\Rightarrow$ <u>Can</u> use fewer equations.

2. (1 point) Is the following statement true or false? Explain your reasoning. "If $\dim V = p$, then there exists a spanning set of $p + 1$ vectors in V."

True. Let $\{v_1, \ldots, v_p\}$ be basis for V. Then

$$\text{span}\{v_1, \ldots, v_p, 2v_1\} = V.$$

3. (3 points) (a) Is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ an eigenvector for $\begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix}$? Show your work.

(b) Is 10 an eigenvalue for $\begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix}$? Show your work.

(a) $\begin{bmatrix} 4 & -2 \\ -3 & 9 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ so $\boxed{\text{yes}}$.

(b) $\begin{bmatrix} 4-10 & -2 \\ -3 & 9-10 \end{bmatrix} = \left[\begin{array}{cc|c} -6 & -2 & 0 \\ -3 & -1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} -3 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

So yes 10 is an eigenvalue, b/c has free variables.

4. (2 points) Determine the dimensions of Nul A and Col A for $A = \begin{bmatrix} 1 & 0 & 9 & 1 & 2 \\ 0 & 0 & 1 & -4 & 7 \end{bmatrix}$.

dim (Nul A) = # non pivots = 3

dim (Col A) = # pivots = 2

Extra Credit(1 point): By inspection, find a nonzero vector in Nul A, where $A = \begin{bmatrix} 1 & 0 & 9 & 1 & 2 \\ 0 & 0 & 1 & -4 & 7 \end{bmatrix}$.

$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ is in Nul A.